

CALCULATION OF FINITE DIFFERENCE METHOD AND DIFFERENTIAL
QUADRATURE METHOD FOR BURGERS EQUATION

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*To my beloved parents Ab Aziz B. Ismail, Rohaya Bt Jusoh @ Ghazali
my wife, Hazwani Bt Abu Bakar @ Hassan, my little girl, Aina Qaisara
and all of my dear friends who supported me.*

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ABSTRACT

In this study, the Finite Difference Method and Differential Quadrature Method are used to solve the Burgers equation. These methods are used to solve some examples of Burgers equation. The different number of nodes is used in these methods to investigate in terms of accuracy study. The solutions of these methods are compared in terms of accuracy of the numerical solution. C- language programs have been developed based on the discussion in order to solve the Burgers equation. The results of this study are collected to compare the solution in terms of convergence study and the accuracy of the numerical solution. The different number of nodes also can affect the solution in term of accuracy study. Decreasing the number of nodes will increasing the errors of the solution. Generally, from the results between the Finite Different Method and Differential Quadrature Method showed the Differential Quadrature Method is better than the Finite Different Method in terms of accuracy of the numerical solution and in terms of convergence study.

ABSTRAK

Di dalam pengajian ini, FDM dan DQM digunakan untuk menyelesaikan masalah persamaan Burgers. Kaedah-kaedah ini digunakan untuk menyelesaikan beberapa contoh persamaan Burgers. Perbezaan saiz jarak titik x digunakan dalam kaedah-kaedah ini untuk mengkaji dari segi ketepatan jawapan. Penyelesaian kaedah-kaedah ini dibandingkan dari segi ketepatan jawapan daripada penyelesaian secara teori. Bagi menyelesaikan masalah ini, program bahasa C telah digunakan. Kemudian data daripada setiap kaedah dikumpulkan dan dibandingkan dengan jawapan sebenar untuk mengkaji dari segi ketepatan jawapan. Semakin kecil saiz jarak titik akan menyebabkan jawapan semakin tidak tepat. Secara keseluruhannya, daripada keputusan dan data yang telah dikumpulkan, didapati kaedah DQM lebih memberikan ketepatan jawapan berbanding dengan kaedah FDM.

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LIST OF ABBREVIATIONS

FDM	- Finite Difference Method
PDE	- Partial Differential Equation
DQM	- Differential Quadrature Method
RK	- Runge-Kutta
ODE	- Ordinary Differential Equation

CHAPTER 1

INTRODUCTION

1.1 Background of the Problem

Single or a system of Partial Differential Equations (PDEs) is mostly encountered by us in many sciences and engineering fields. PDEs also describe many of the basic natural laws in physical or chemical phenomena. In this study, the partial differential equation considered is the Burgers–Huxley equation which can effectively models the interaction between reaction mechanisms, convection effects and diffusion transports (Murat Sari and Gurhan Gurarslan, 2009). In general, the closed–form solution is not available or not easily obtained because most of these problems may involved the nonlinear partial differential equations. This fact leads to the development of another alternative to approximate the solutions of these partial differential equations. As a result, after years of researches scientists found that the approximation of the solution of the system of partial differential equations can be obtained by using numerical discretization techniques on some function value at certain discrete points, so-called grid points or mesh points. There are three most commonly used numerical methods in engineering and in computational fluid dynamics are the finite difference, finite element, and the finite volume methods.

One of the techniques to solve the Burgers equation is by using the Finite Difference Method (FDM) which is the simplest method. This method solved by replacing the values at certain grid points and approximates the derivatives by differences in these values. The partial derivatives in the PDE at each grid point are approximated from the neighborhood values.

Another technique which is discussed in this study is Differential Quadrature Method (DQM). As stated by (C. Shu, 2000), DQM is an extension of FDM for the highest order of finite difference scheme. This method represents by sum up all the derivatives of the function at any grid points, and then the equation transforms to a system of ordinary differential equations (ODEs) or a set of algebraic equations (R.C. Mittal and Ram Jiware, 2009). The system of ordinary differential equations is then solved by numerical methods such as the implicit Runge-Kutta method that will be discussed in order to get the solutions in this study.

1.2 Statement of the Problem

There are many ways to approximate the solution of Burgers equation. A set of initial and boundary conditions are needed to solve the Burgers equation. Finite Difference Method (FDM) is easy to solve the examples of Burgers equation but is less accurate. Then, the Differential Quadrature method (DQM) is used to solve the problem but it needs more calculation and more time.

1.3 Objectives of the Study

The objectives of this study are:

- i. To solve Burgers equation numerically using FDM and DQM.
- ii. To compare the FDM and DQM in terms of accuracy of Burgers equation.
- iii. To develop C language program codes for FDM and DQM.

1.4 Scope of the Study

In this study, the main numerical technique discussed is the Differential Quadrature Method and Finite Differential method. The accuracy or convergence on DQM in solving the 1 D Burgers equation with Dirichlet's boundary conditions will be discussed in this study. Other than that, the results from DQM and FDM will be used to compare with the exact solutions in term of the accuracy of numerical solution in solving the Burgers equation. In this study also focused on solving the Burgers equation.

1.5 Significance of the study

In this study, the FDM and DQM will be discussed and applied to Burgers equation which is important in engineering field. This project also will give benefit to other researchers in this area of linear and nonlinear partial differential equation to be able to understand how the methods work on solving the Burgers equation with dirichlet boundary conditions.

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APPENDIX A

The calculation of the exact for Example 4.1.

```
> restart;
> pde := diff(u(x, t), t) - 1·diff(diff(u(x, t), x), x) + u(x, t)
    ·diff(u(x, t), x) = 0;

$$pde := \frac{\partial}{\partial t} u(x, t) - \left( \frac{\partial^2}{\partial x^2} u(x, t) \right) + u(x, t) \left( \frac{\partial}{\partial x} u(x, t) \right) = 0$$

> ibc := {u(x, 0) = sin(2·π·x), u(0, t) = 0, u(1, t) = 0};

$$ibc := \{u(0, t) = 0, u(1, t) = 0, u(x, 0) = \sin(2 \pi x)\}$$

> pds := pdsolve(pde, ibc, numeric, time = t, range = 0..1);
    pds := module( )
        export plot, plot3d, animate, value, settings;
        ...
    end module

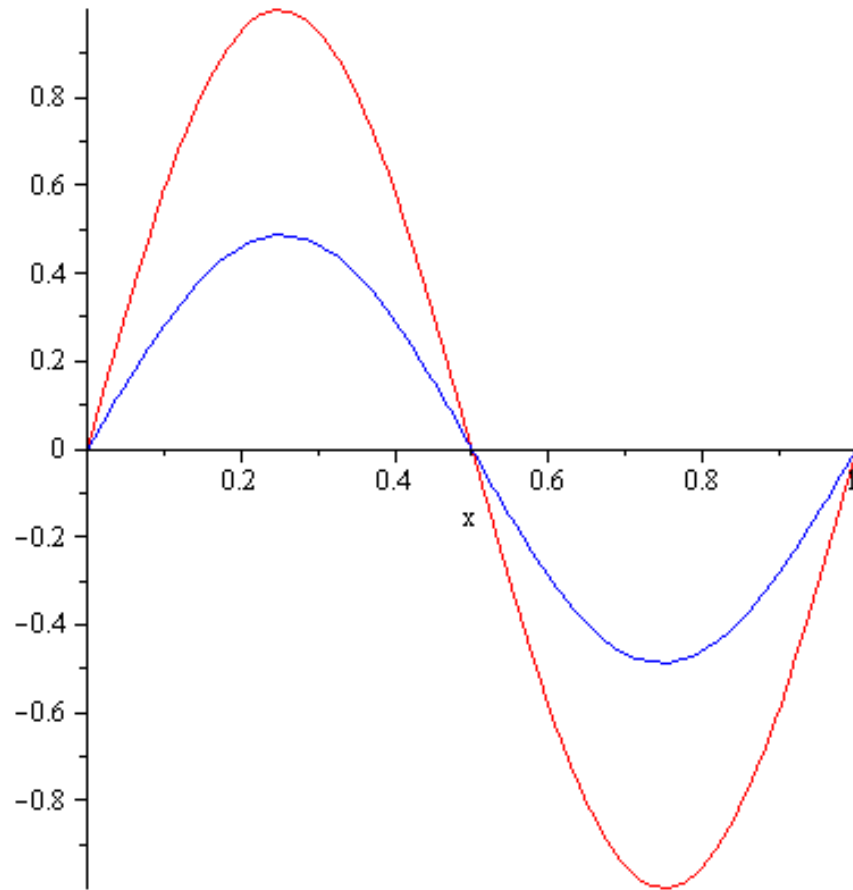
> pa := pds:-plot(t = 0, numpoints = 50);

$$pa := PLOT(\dots)$$

> pb := pds:-plot(t = 0.02, numpoints = 50, color = blue);

$$pb := PLOT(\dots)$$

> plots[display]( {pa, pb} );
```



```

> pds:-value(t=0.02,output = listprocedure );
      [x = proc(x) ... end proc, t = 0.02, u(x, t) = proc(x) ... end proc]

> uval := rhs(op(3, %));
      uval := proc(x) ... end proc

> pds1 := pdsolve( pde, ibc, numeric, 'spacestep' = 1/40, time = t, range
      = 0..1 );

      pds1 := module( )
      export plot, plot3d, animate, value, settings;
      ...
      end module

> uval(0.8); ### fsolve(uval(x) = 0.05, x = 0..1);
      -0.45944697252221633

```

APPENDIX B

The result from Example 4.1

Number of nodes , $N = 10$

$u(\mathbf{x}, t)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.587785	0.951057	0.951057	0.587785	0	-0.58779	-0.95106	-0.95106	-0.58779	0
0.002	0	0.538305	0.875908	0.881728	0.547908	0	-0.54791	-0.88173	-0.87591	-0.53831	0
0.004	0	0.493952	0.807401	0.816634	0.509462	0	-0.50946	-0.81664	-0.8074	-0.49395	0
0.006	0	0.453904	0.744724	0.755779	0.472794	0	-0.47279	-0.75578	-0.74472	-0.4539	0
0.008	0	0.41759	0.68722	0.699056	0.438111	0	-0.43811	-0.69906	-0.68722	-0.41759	0
0.01	0	0.384574	0.634356	0.646298	0.40551	0	-0.40551	-0.6463	-0.63436	-0.38457	0
0.012	0	0.354494	0.585696	0.597308	0.375008	0	-0.37501	-0.59731	-0.5857	-0.35449	0
0.014	0	0.327035	0.540868	0.551875	0.346572	0	-0.34657	-0.55188	-0.54087	-0.32704	0
0.016	0	0.301916	0.499546	0.509785	0.320131	0	-0.32013	-0.50979	-0.49955	-0.30192	0
0.018	0	0.278892	0.461441	0.470827	0.295597	0	-0.2956	-0.47083	-0.46144	-0.27889	0
0.02	0	0.257745	0.426292	0.434795	0.272868	0	-0.27287	-0.4348	-0.42629	-0.25775	0

Number of nodes , $N = 20$

	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
0	0	0.309017	0.587785	0.809017	0.951057	1	0.951057	0.809017	0.587785	0.309017	0
0.002	0	0.282666	0.538469	0.742876	0.875898	0.924026	0.881731	0.752315	0.547907	0.2885	0
0.004	0	0.259325	0.494518	0.683359	0.807408	0.853775	0.81664	0.698299	0.50946	0.26856	0
0.006	0	0.238443	0.455015	0.629477	0.744812	0.788859	0.755793	0.647249	0.472792	0.249429	0
0.008	0	0.219623	0.419274	0.580458	0.687459	0.728889	0.699093	0.599289	0.438111	0.231253	0
0.01	0	0.202586	0.386768	0.535687	0.634801	0.67349	0.646381	0.554435	0.405516	0.214115	0
0.012	0	0.187223	0.357115	0.494656	0.586373	0.622315	0.597462	0.512627	0.375028	0.198047	0
0.014	0	0.173753	0.33022	0.456904	0.541764	0.575045	0.55212	0.473759	0.346614	0.183049	0
0.016	0	0.162712	0.307316	0.421874	0.500547	0.531402	0.51014	0.437693	0.320206	0.169095	0
0.018	0	0.149407	0.296954	0.388624	0.46189	0.491207	0.471329	0.404265	0.295711	0.156144	0
0.02	0	0.066224	0.347458	0.356375	0.422343	0.454544	0.43566	0.373275	0.273018	0.144151	0

0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
-0.30902	-0.58779	-0.80902	-0.95106	-1	-0.95106	-0.80902	-0.587785	-0.309017	0
-0.2885	-0.54791	-0.75232	-0.88173	-0.92403	-0.8759	-0.74288	-0.538469	-0.282666	0
-0.26856	-0.50946	-0.6983	-0.81664	-0.85378	-0.80741	-0.68336	-0.494518	-0.259324	0
-0.24943	-0.47279	-0.64725	-0.75579	-0.78886	-0.74481	-0.62948	-0.455016	-0.23844	0
-0.23125	-0.43811	-0.59929	-0.69909	-0.72889	-0.68746	-0.58046	-0.419274	-0.219606	0
-0.21412	-0.40552	-0.55444	-0.64638	-0.67349	-0.6348	-0.53569	-0.386761	-0.202499	0
-0.19805	-0.37503	-0.51263	-0.59746	-0.62231	-0.58637	-0.49469	-0.357037	-0.186805	0
-0.18305	-0.34661	-0.47376	-0.55212	-0.57503	-0.54179	-0.4571	-0.329594	-0.172007	0
-0.16909	-0.32021	-0.4377	-0.51013	-0.53132	-0.50075	-0.42276	-0.302949	-0.157069	0
-0.15614	-0.29571	-0.4043	-0.47126	-0.49082	-0.46341	-0.39217	-0.26912	-0.143956	0
-0.14414	-0.27304	-0.37344	-0.43513	-0.45287	-0.43239	-0.3667	-0.182275	-0.18866	0